Copula theory and applications: Part III Applications

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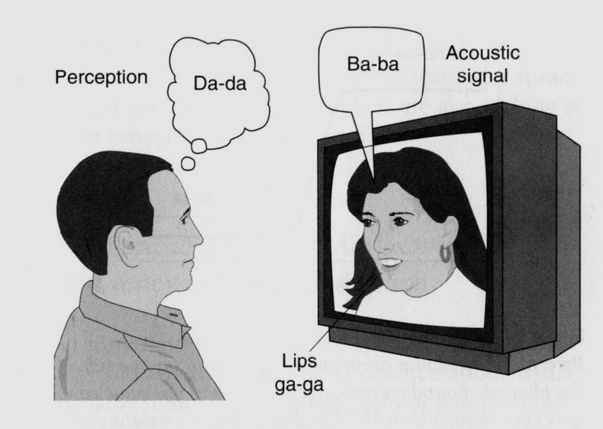
Outline

Multisensory integration: reaction time modeling

Non-linear dependency

Stop signal paradigm: reaction time modeling

McGurk-MacDonald effect



Link McGurk H, Macdonald J 1976 Hearing lips and seeing voices. *Nature*, 264, 746–748

Defining multisensory integration

- **Multisensory integration (MI)** “Inputs from two or more senses are combined to form a product that is distinct from, and thus cannot be easily “deconstructed” to reconstitute, the components from which it is created”(Stein & Meredith 1993)

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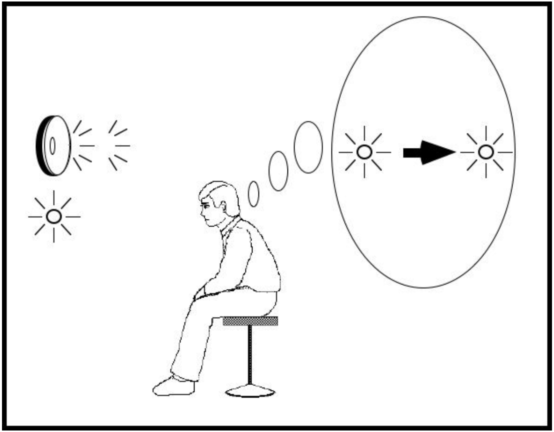
- **Crossmodal interaction** is defined as the situation in which the perception of an event as measured in terms of one modality is changed in some way by the concurrent stimulation of one or more other sensory modalities (Welch and Warren 1986).

  Multisensory integration: a putative mechanism generating *crossmodal interaction* (level: sensory, cognitive, and/or motor)

Sound-Induced Flash Illusion



From: http://www.psy.l.chiba-u.ac.jp/labo/vision2/SIF.html ▸ Link Shams, Kamitani; and Shimojo (2000) Ilusions. What you see is what you hear. *Nature*, 408, 788

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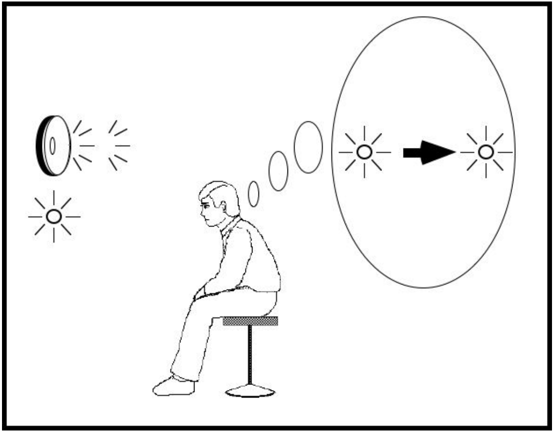

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Cochlear implant patients have been shown to be more susceptible to the illusion.

Multisensory RT paradigm: Todd 1912

REACTION TO MULTIPLE STIMULI

BY

JOHN WELHOFF TODD, Ph.D.

ARCHIVES OF PSYCHOLOGY

EDITED BY

R. S. WOODWORTH

No. 25, August, 1912

COLUMBIA CONTRIBUTIONS TO PHILOSOPHY AND PSYCHOLOGY
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THE SCIENCE PRESS

CONTENTS

Multisensory RT paradigm: Todd 1912

Fig. 4. Graphic Representation of Table XI.

L: light – E: shock – S: sound

Measuring multisensory integration in RTs

- experimental conditions:

  V – only a visual stimulus is presented
  A – only an auditory stimulus is presented
  VA – a visual-auditory stimulus pair is presented
  Task: Respond as quickly as possible to a stimulus of any modality ("redundant signals/targets paradigm")

Race model inequality

- aka "Miller's inequality" (Miller 1982)
- For VA experiment:

$$P_{VA}(\min\{V, A\} \leq t) \leq P_V(V \leq t) + P_A(A \leq t)$$

Race model inequality

$$F_{VA}(t) \leq \underbrace{\min\{F_V(t) + F_A(t), 1\}} \quad (*)$$

The upper bound is the distribution function of random variable $\min(\hat{V}, \hat{A})$ with **maximal negative dependence** between $\hat{V}$ and $\hat{A}$!

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This follows easily from the (lower of the) *Fréchet-Hoeffding bounds* for any bivariate distribution (Colonius, 1990),

$$\max\{F_V(s) + F_A(t) - 1, 0\} \leq H_{VA}(s,t) \leq \min\{F_V(s), F_A(t)\}. \quad (1)$$

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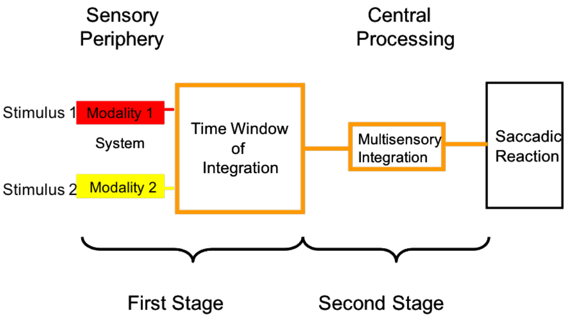
- but it could also be evidence against the "context invariance" assumption
- The upper (lower) bound in (1) is known as "comonotonicity" ("countermonotonicity") **copula**.

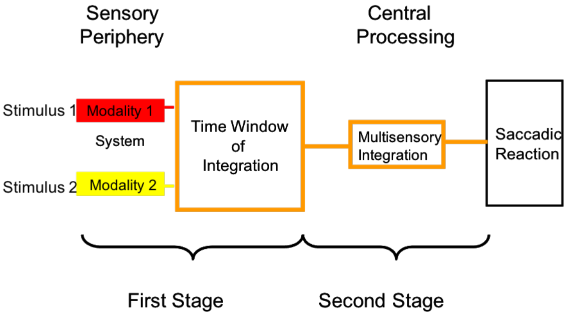
“Time window of integration” modeling framework



Cartoon version of TWIN framework

- First stage: race among the peripheral processes in the sensory (V, A, T) pathways triggered by a crossmodal stimulus complex.

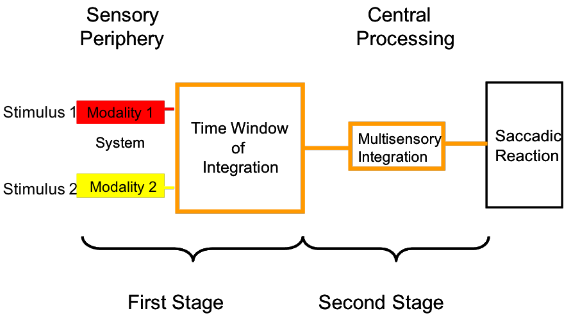
“Time window of integration” modeling framework



Cartoon version of TWIN framework

- Time-Window assumption: multisensory integration occurs **only** if the peripheral processes of the first stage all terminate within a given temporal interval, the “time window of integration”.

“Time window of integration” modeling framework



Cartoon version of TWIN framework

- Second stage: all processes following the first stage including preparation and execution of a response.

Model assumptions

(i) V and A peripheral processing times for visual and auditory stimuli in the first stage

(ii) (W_1, W_2) random vector, $W_1 = \min(A + \tau, V)$
with $\tau = SOA$ and
W_2 random duration of the second stage

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(iii) T (observable) reaction time in the auditory-visual condition:

$$T = W_1 + W_2$$

(iv) **time-window assumption**: necessary for integration (I) to occur:

$$I(\omega, \tau) = \{\max(A + \tau, V) < \min(A + \tau, V) + \omega\},$$

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(v) **stochastic independence assumption**: $W_1|I$ and $W_2|I$ are *conditionally independent*

Expected reaction time

- $\pi = \mathbb{P}[I]$ prob. of integration and $1 - \pi = \mathbb{P}[C]$ prob. of no integration

$$\begin{aligned} \mathrm{E}[T] &= \mathrm{E}[W_1 + W_2] \\ &= \pi \, \mathrm{E}[W_1 + W_2 | I] + (1 - \pi) \, \mathrm{E}[W_1 + W_2 | C] \\ &= \mathrm{E}[W_1 + W_2 | C] - \pi \times (\Delta_1 + \Delta_2) \end{aligned}$$

where

$$\Delta_i = \mathrm{E}[W_i | C] - \mathrm{E}[W_i | I], \quad i = 1, 2$$

the magnitude of the integration effect in stage i.

TWIN model: Structure of dependence between the stages

Going beyond the mean

$$T = W_1 + W_2$$

W_1 and W_2 conditionally independent (on I)

$$\mathrm{V}[T] = \mathrm{V}[W_1 + W_2] = \mathrm{V}[W_1] + \mathrm{V}[W_2] + 2\,\mathrm{Cov}[W_1, W_2]$$

What can we say about $\mathrm{Cov}[W_1, W_2]$ w/o assuming specific distributions?

What about non-linear dependency between W_1 and W_2 ?

Going beyond the mean: $\mathrm{Cov}[W_1, W_2]$

Proposition 1

$$\mathrm{Cov}[W_1, W_2] = \pi(1-\pi)\,\Delta_1\,\Delta_2$$

where

$$\Delta_1 = \mathrm{E}[W_1|C] - \mathrm{E}[W_1|I] \text{ and } \Delta_2 = \mathrm{E}[W_2|C] - \mathrm{E}[W_2|I].$$

Thus, for π different from zero or one,
- $\mathrm{Cov}[W_1, W_2]$ is **positive** if for both W_1 and W_2 the mean is (strictly) larger, or smaller, under the event of "no integration" than under "integration" (facilitation or inhibition).

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- $\mathrm{Cov}[W_1, W_2]$ is **zero** if there is no effect of integration: $\Delta_1 = 0$ or $\Delta_2 = 0$.

The variance of W_1, W_2

For $i = 1, 2$,

$$\mathrm{V}[W_i] = \pi \mathrm{V}[W_i | I] + (1 - \pi)\mathrm{V}[W_i | C] + \pi(1 - \pi)\Delta_i^2.$$

- weighted average of the conditional variances (weighted by the probability of I and C occurring)
- plus an additional, non-negative term $\pi(1 - \pi)\Delta_i^2$ that is due to the effect of the mixture generated by the occurrence of I or C, and is maximal for $\pi = 0.5$

Non-linear dependency of (W_1, W_2): Kendall's τ

- Let (W_{i1}, W_{i2}), $i = 1, 2$, be two independent and identically distributed vectors with joint distribution function H.

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  At the population level, Kendall's tau is defined as the probability of "concordance" between the two vectors minus the probability of "discordance", i.e.,

$$\begin{aligned}\tau(W_1, W_2) =& \mathbb{P}[(W_{11} - W_{21})(W_{12} - W_{22}) > 0] \\ & - \mathbb{P}[(W_{11} - W_{21})(W_{12} - W_{22}) < 0)]\end{aligned}$$

  e.g., W_{11} is the first-stage processing time in $i = 1$ and W_{21} is the first-stage processing time in $i = 2$.

Kendall's τ for TWIN model

Proposition 2

$$\tau(W_1, W_2) = 2\,\pi(1-\pi)(2V-1)$$

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Proposition 3

For π different form zero or 1,
(i) if $(W_1|C) \stackrel{d}{=} (W_1|I)$ or $(W_2|C) \stackrel{d}{=} (W_2|I)$, then $\tau(W_1, W_2) = 0$.

(ii) $\tau(W_1, W_2) \geq -0.5$.

Summing up dependency results

(i) Kendall's τ is non-zero only if both pairs of marginals, (F_I, F_C) and (G_I, G_C) contain nonidentical distributions. That is, there must be an effect of integration in both stages.

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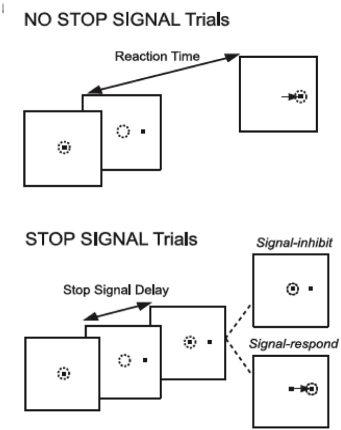
- Additional result: the **sign** of dependency is determined by

$$[W_1|I] \leq_{lr} [W_1|C] \quad \text{and } [W_2|I] \leq_{lr} [W_2|C]$$

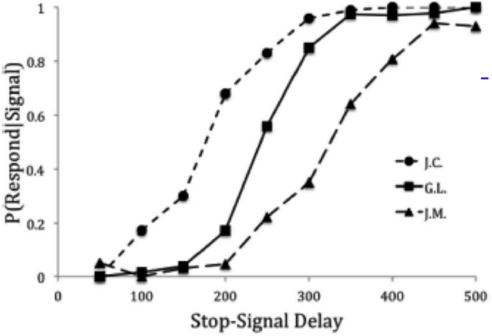
$\leq_{lr}$ *likelihood ratio order* (aka *totally positive of order 2, TP_2*)

Response Inhibition: Stop signal paradigm



- Subjects are instructed to make a response as quickly as possible to a go signal (no-stop-signal trial)
- On a minority of trials, a stop signal is presented and subjects have to inhibit the previously planned response (stop-signal trial)

Stop signal paradigm: inhibition functions



- Inhibition functions of three subjects (Logan & Cowan, 1984)
- The inhibition function is determined by stop-signal delay, but it also depends strongly on RT in the go task; the probability of responding given a stop signal is lower the longer the go RT

The general race model

The general race model (1)

- Distinguish two different experimental conditions termed "context GO," where only a go signal is presented, and "context STOP", where a stop signal is presented in addition.

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- In STOP, let T_{go} and T_{stop} denote the random processing time for the go and the stop signal, respectively, with (unobservable !) bivariate distribution function

$$H(s,t) = \mathbb{P}(T_{go} \leq s,\, T_{stop} \leq t),$$

for all $s, t \geq 0$.

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$$H(s,t) = \mathbb{P}(T_{go} \leq s, T_{stop} \leq t),$$

  for all $s, t \geq 0$.
- The marginal distributions of $H(s,t)$ are denoted as

$$F_{go}(s) = \mathbb{P}(T_{go} \leq s, T_{stop} < \infty)$$
$$F_{stop}(t) = \mathbb{P}(T_{go} < \infty, T_{stop} \leq t).$$

The general race model (2)

NOTE: The distribution of T_{go} could be different in context GO and in context STOP. However, the general race model rules this out by adding the important

> **Context invariance assumption** *The distribution of go signal processing time T_{go} is the same in context GO and context STOP.*

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Race assumption Probability of a response despite stop signal at delay t_d:

$$p_r(t_d) = \mathbb{P}(T_{go} < T_{stop} + t_d) \tag{2}$$

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Assume $H(s,t) = \mathbb{P}(T_{go} \leq s, T_{stop} \leq t)$ is absolutely continuous, so that bivariate and marginal densities exist.

The general race model (3)

The RT distribution of responses given a stop signal at delay t_d (signal-response distribution) is

$$F_{sr}(t \mid t_d) = \mathbb{P}(T_{go} \leq t \mid T_{go} < T_{stop} + t_d)$$

Goal: Estimate the **unobservable** stop-signal processing time distribution $F_{stop}(t)$ as a measure of control behavior.

The independent race model

The **independent** race model (1)

Logan & Cowan (1984) suggested the *independent race model* assuming stochastic independence between T_{go} and T_{stop}:

Stochastic independence: for all s, t

$$H(s,t) = \mathbb{P}(T_{go} \leq s)\,\mathbb{P}(T_{stop} \leq t) = F_{go}(s)\,F_{stop}(t)$$

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Then

$$\begin{aligned} p_r(t_d) &= \mathbb{P}(T_{go} < T_{stop} + t_d) \\ &= \int_0^{\infty} f_{go}(t)\,[1 - F_{stop}(t - t_d)]\,\mathrm{d}t. \end{aligned} \tag{3}$$

The **independent** race model (2)

Density of the signal-response time distribution $F_{sr}(t|t_d)$, for $t > t_d$

$$f_{sr}(t \mid t_d) = f_{go}(t)\,[1 - F_{stop}(t - t_d)]/p_r(t_d).$$

The **independent** race model (2)

Density of the signal-response time distribution $F_{sr}(t|t_d)$, for $t > t_d$

$$f_{sr}(t \mid t_d) = f_{go}(t)\,[1 - F_{stop}(t - t_d)]/p_r(t_d).$$

Unfortunately obtaining reliable estimates for the stop signal distribution requires unrealistically large numbers of observations in practice (Band et al. 2003; Matzke et al. 2013).

The **independent** race model (3)

- integration method
- mean method

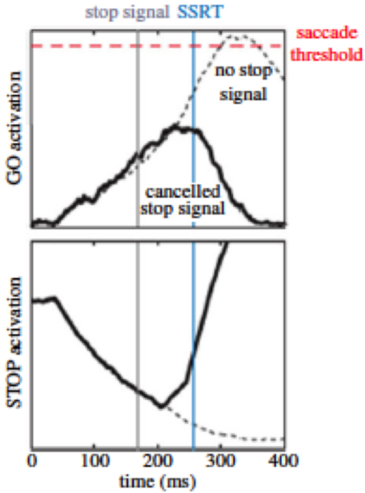
see:

A consensus guide to capturing the ability to inhibit actions and impulsive behaviors in the stop-signal task. **Verbruggen F**, Aron AR, Band GP, Beste C, Bissett PG, Brockett AT, Brown JW, Chamberlain SR, Chambers CD, Colonius H, Colzato LS, Corneil BD, Coxon JP, Dupuis A, Eagle DM, Garavan H, Greenhouse I, Heathcote A, Huster RJ, Jahfari S, Kenemans JL, Leunissen I, Li CR, Logan GD, Matzke D, Morein-Zamir S, Murthy A, Paré M, Poldrack RA, Ridderinkhof KR, Robbins TW, Roesch M, Rubia K, Schachar RJ, Schall JD, Stock AK, Swann NC, Thakkar KN, van der Molen MW, Vermeylen L, Vink M, Wessel JR, Whelan R, Zandbelt BB, Boehler CN.
Elife. 2019 Apr 29;8. pii: e46323. doi: 10.7554/eLife.46323.

The paradox

A paradox

Studying saccade countermanding in monkeys, Hanes and colleagues (Hanes & Schall 1995, Hanes et al. 1998) recorded from frontal and supplem. eye fields. They found neurons involved in gaze-shifting and gaze-holding that modulate on stop-signal trials, just before SSRT when the monkey stopped successfully. (Figure from Schall & Logan 2017)



A paradox

- The paradox: How can interacting circuits of mutually inhibitory gaze-holding and gaze-shifting neurons instantiate STOP and GO processes with independent finishing times?

Definition: Bivariate Copula ($n = 2$)

Definition

An 2-*copula* is a bivariate distribution function C with univariate margins uniformly distributed on $[0, 1]$.

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Theorem (Sklar's Theorem, 1959)

Let $F(x_1, x_2)$ *be a bivariate distribution function with margins* $F_1(x_1), F_2(x_2)$*; then there exists an* 2*-copula* $C : [0, 1]^2 \longrightarrow [0, 1]$ *that satisfies*

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)), \quad (x_1, x_2) \in \Re^2.$$

If F_1^{-1}, F_2^{-1} *are the quantile functions of the margins, then for any* $(u_1, u_2) \in [0, 1]^2$

$$C(u_1, u_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2)).$$

Example 1: $C(u_1, u_2) = u_1 u_2 = F_{go}(s)\, F_{stop}(t)$ (independence copula)

Example 2: The bivariate Farlie-Gumbel-Morgenstern (FGM) copula

$$C_\delta(u_1, u_2) = u_1\, u_2[1 + \delta(1 - u_1)(1 - u_2)],$$

for $-1 \leq \delta \leq 1$ and $0 \leq u_1, u_2 \leq 1$.

Resolving the paradox:
a race model with perfect negative dependence

The Fréchet-Hoeffding lower bound copula

- For any bivariate distribution function

$$H(s,t) = \mathbb{P}(T_{go} \leq s, T_{stop} \leq t)$$

the following inequality holds:

$$H^{-}(s,t) \leq H(s,t)$$

with $H^{-}(s,t) = \max\{F_{go}(s) + F_{stop}(t) - 1, 0\}$

Perfect negative dependence

What does it mean?

$$H^{-}(s,t) = \max\{F_{go}(s) + F_{stop}(t) - 1, 0\}. \quad (4)$$

for all s, t $(s, t \geq 0)$.

The marginal distributions of $H^{-}(s,t)$ are the same as before, that is, $F_{go}(s)$ and $F_{stop}(t)$.

Perfect negative dependence

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$$F_{stop}(T_{stop}) = 1 - F_{go}(T_{go}) \quad (5)$$

holds "almost surely", that is, with probability 1.

Perfect negative dependence: the key property

$$F_{stop}(T_{stop}) = 1 - F_{go}(T_{go}) \qquad (6)$$

holds "almost surely", that is, with probability 1.

- Thus, for any F_{go} percentile we immediately obtain the corresponding F_{stop} percentile as complementary probability and vice versa, which expresses perfect negative dependence between T_{go} and T_{stop}.

Perfect negative dependence: the key property

$$F_{stop}(T_{stop}) = 1 - F_{go}(T_{go}) \tag{6}$$

holds "almost surely", that is, with probability 1.

- Thus, for any F_{go} percentile we immediately obtain the corresponding F_{stop} percentile as complementary probability and vice versa, which expresses perfect negative dependence between T_{go} and T_{stop}.
- It constitutes the most direct implementation of the notion of "mutual inhibition" observed in neural data: any increase of inhibitory activity (speed-up of T_{stop}) elicits a corresponding decrease in "go" activity (slow-down of T_{go}) and vice versa.

Can we test for PND ?

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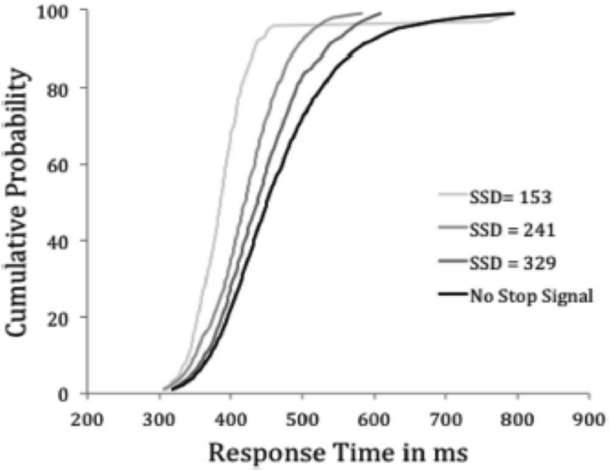
"Fan effect":



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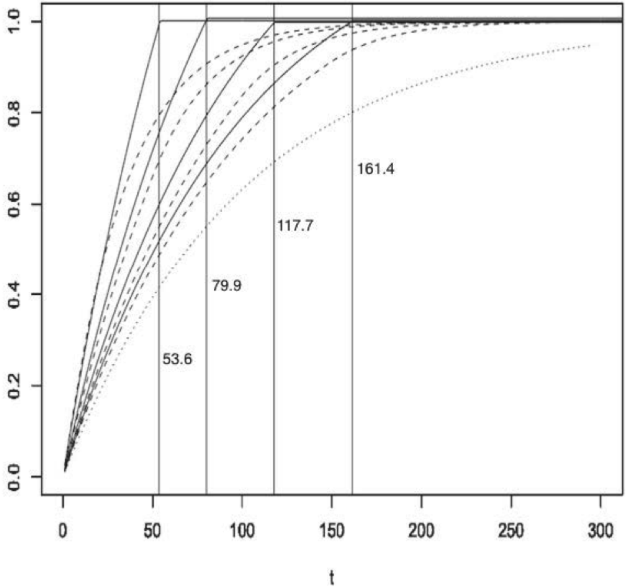

dashed = IND
line = PND
*T_{go}, T_{stop}: exponential distribution
*simulation: **copBasic** package in **R**

Predictions from perfect negative dependence

Do we have to throw away all measures obtained using the independent model, like estimates of mean $T_{stop} \equiv \text{SSRT}$?

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No ! Because the (marginal) distribution of T_{stop} are the same under independence and perfect negative dependence. Thus

$$\mathrm{E}[T_{stop} \,|\, \mathrm{IND}] = \mathrm{E}[T_{stop} \,|\, \mathrm{PND}]$$

- Colonius H, Diederich A (2018) Paradox resolved: stop signal race model with negative dependence. *Psychological Review,* Vol. 125, No. 6, 1051–1058.

Race models with moderate (negative) dependence

Moderate dependence: the bivariate FGM copula

$$C_\delta(u_1, u_2) = u_1\, u_2[1 + \delta(1 - u_1)(1 - u_2)],$$

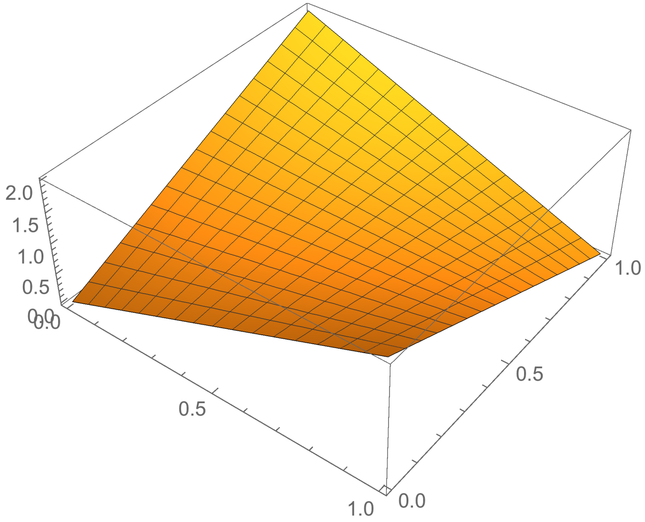
for $-1 \leq \delta \leq 1$

$$\begin{aligned} C_\delta(F_{go}(s), F_{stop}(t)) =& \mathbb{P}[T_{go} \leq s, T_{stop} \leq t] \\ =& F_{go}(s)F_{stop}(t)[1 + \delta(1 - F_{go}(s))(1 - F_{stop}(t))] \end{aligned}$$

Kendall's tau:

$$\tau(T_{go}, T_{stop}) = \frac{2\delta}{9} \geq -0.222.$$

Bivariate FGM copula with $\delta = -1, \tau = -0.222$



Dependent race model with FGM copula

$$\mathbb{P}[T_{go} \leq s, T_{stop} \leq t]$$
$$= F_{go}(s)F_{stop}(t)[1 + \delta(1 - F_{go}(s))(1 - F_{stop}(t))].$$

- Task: estimate δ and $F_{stop}(t)$ from observables ! Work in progress...

For example,

$$\mathbb{P}(T_{go} \leq t | T_{stop} = t - t_d) = F_{go}(t)\,[1 + \delta(1 - F_{go}(t))(1 - 2F_{stop}(t - t_d))].$$

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- Race models with **moderate stochastic dependency** can be constructed via copulas but...
- ...efficient estimation methods and non-arbitrary choice of the copula type need further investigation.

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